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**THE SCALAR $q\bar{q}$ NONET AND
CONFIRMATION OF THE BROAD $\sigma(\approx 500)$ MESON^a**

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The available data on the $a_0(980)$, $f_0(980)$, $f_0(1300)$ and $K_0^*(1430)$ mesons are fitted as a distorted 0^{++} $q\bar{q}$ nonet using only 6 parameters and a very general model. This includes all light two-pseudoscalar thresholds, constraints from Adler zeroes, flavour symmetric couplings, unitarity and physically acceptable analyticity. One finds that with the large overall coupling there can appear two physical resonance poles from only one $q\bar{q}$ state. Thus the $f_0(980)$ and $f_0(1300)$ resonance poles are two manifestations of the same $s\bar{s}$ state. On the other hand, the $u\bar{u} + d\bar{d}$ state, when unitarized and strongly distorted by hadronic mass shifts, becomes an extremely broad Breit-Wigner-like background, $m_{BW} = 860$ MeV, $\Gamma_{BW} = 880$ MeV, with its pole at $s=(0.158-i0.235)$ GeV². This is the σ meson required by models for spontaneous breaking of chiral symmetry.

1 Introduction

This paper is a short summary of two recent papers^{1,2}, including a few new comments. These results give a new understanding of the controversial light scalar mesons. It is shown that one can describe the S-wave data on the light $q\bar{q}$ nonet with a model which includes most well established theoretical constraints: Adler zeroes as required by chiral symmetry, all light two-pseudoscalar (PP) thresholds with flavor symmetric couplings, physically acceptable analyticity, and unitarity. A unique feature of this model is that it simultaneously describes the whole scalar nonet and one obtains a good representation of a large set of relevant data. Only six parameters, which all have a clear physical interpretation, are needed: an overall coupling constant ($\gamma = 1.14$), the bare mass of the $u\bar{u}$ or $d\bar{d}$ state ($m_0 = 1.42$ GeV), the extra mass for a strange quark ($m_s - m_u = 100$ MeV), a cutoff parameter ($k_0 = 0.56$ GeV/c), an Adler zero parameter for $K\pi$ ($s_{A_{K\pi}} = -0.42$ GeV), and a phenomenological parameter enhancing the $\eta\eta'$ couplings ($\beta = 1.6$).

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2 Understanding the S-waves

In Figs. 1-3 we show the obtained fits to the $K\pi$, $\pi\pi$ S-waves and to the $a_0(980)$ resonance peak in $\pi\eta$. The partial wave amplitude is in the case of one $q\bar{q}$ resonance, such as the $a_0(980)$ can be written

$$A(s) = -Im\Pi_{\pi\eta}(s)/[m_0^2 + Re\Pi(s) - s + iIm\Pi(s)], \quad (1)$$

where

$$Im\Pi(s) = \sum_i Im\Pi_i(s) \quad (2)$$

$$= -\sum_i \gamma_i^2 (s - s_{A,i}) \frac{k_i}{\sqrt{s}} e^{-k_i^2/k_0^2} \theta(s - s_{th,i}),$$

$$Re\Pi_i(s) = \frac{1}{\pi} \mathcal{P} \int_{s_{th,1}}^{\infty} \frac{Im\Pi_i(s)}{s' - s} ds'. \quad (3)$$

Here the coupling constants γ_i are related by flavour symmetry and the OZI rule, such that there is only one over all parameter γ . The $s_{A,i}$ are the positions of the Adler zeroes, which normally are $s_{A,i} = 0$, except $s_{A,\pi\pi} = m_\pi^2/2$, and $s_{A,K\pi}$, which is a free parameter.

In the flavourless channels the situation is a little more complicated than eqs. (1-3) since one has both $u\bar{u}+d\bar{d}$ and $s\bar{s}$ states, requiring a two dimensional mass matrix (See Ref.¹). Note that the sum runs over all light PP thresholds, which means three for the $a_0(980)$: $\pi\eta$, $K\bar{K}$, $\pi\eta'$ and three for the $K_0^*(1430)$: $K\pi$, $K\eta$, $K\eta'$, while for the f_0 's there are five channels: $\pi\pi$, $K\bar{K}$, $\eta\eta$, $\eta\eta'$, $\eta'\eta'$. Five channels means the amplitudes have $2^5 = 32!$ different Riemann sheets, and in principle there can be poles on each of these sheets. In Fig. 4 we show as an example the running mass, $m_0^2 + Re\Pi(s)$, and the width-like function, $Im\Pi(s)$, for the I=1 channel. The crossing point of the running mass with s gives the 90° mass of the $a_0(980)$. The magnitude of the $K\bar{K}$ component in the $a_0(980)$ is determined by $-\frac{d}{ds} Re\Pi(s)$ which is large in the resonance region just below the $K\bar{K}$ threshold. These functions fix the PWA of eq.(1) and Fig. 3. In Fig. 5 the running mass and width-like function for the strange channel are shown. These fix the shape of the $K\pi$ phase shift and absorption parameters in Fig. 1.

Four out of our six parameters are fixed by the $K\pi$ data leaving only $m_s - m_u = 100$ MeV to "predict" the $a_0(980)$ structure Fig. 3, and the parameter β to get the $\pi\pi$ phase shift right above 1 GeV/c. One could discard the β parameter if one also included the next group of important thresholds or pseudoscalar (0^{-+}) -axial (1^{+-}) thresholds, since then the $K\bar{K}_{1B} + c.c.$

Table 1: Resonances in the S-wave $PP \rightarrow PP$ amplitudes ¹ The first resonance is the σ which we name here $f_0(\approx 500)$. The two following are both manifestations of the same $s\bar{s}$ state. The $f_0(980)$ and $a_0(980)$ have no approximate Breit Wigner-like description, and the Γ_{BW} given for $a_0(980)$ is rather the peak width. The mixing angle δ_S for the $f_0(\approx 500)$ or σ is with respect to $u\bar{u} + d\bar{d}$, while for the two heavier f_0 's it is with respect to $s\bar{s}$.

resonance	m_{BW}	Γ_{BW}	$\delta_{S,BW}$	Comment
$f_0(\approx 500)$	860	880	$(-9 + i8.5)^\circ$	The σ meson.
$f_0(980)$	-	-	-	First near $s\bar{s}$ state
$f_0(1300)$	1186	360	$(-32 + i1)^\circ$	Second near $s\bar{s}$ state
$K_0^*(1430)$	1349	498	-	The $s\bar{d}$ state
$a_0(980)$	987	≈ 100	-	First I=1 state

thresholds give a very similar contribution to the mass matrix as $\eta\eta'$. As can be seen from Figs. 1-3 the model gives a good description of the relevant data.

In Ref.¹ we looked for only those four poles which are nearest to the physical region, and which could complete a multiplet. These were given in Ref.¹: the $f_0(980)$, $f_0(1300)$, $a_0(980)$ and $K_0^*(1430)$. We found parameters for these close to the conventional lightest scalars in the 1994 PDG tables.

3 One $q\bar{q}$ pole can give rise to two resonances

However, in addition there are other image poles³, usually located far from the physical region. As explained in Ref.², and below, some of these can come so close to the physical region that they make new resonances. And, in fact, there are more than four physical poles with different isospin, in the output spectrum of our model, although only four bare states are put in! In Table 2 we list the significant pole positions.

All these poles are manifestations of *the same nonet*². The $f_0(980)$ and the $f_0(1300)$ turn out to be two manifestations of the same $s\bar{s}$ state. Fig. 6 shows how this can come about for the $s\bar{s}$ channel. There can be two crossings with the running mass. Similarly the $a_0(980)$ and the $a_0(1450)$ are likely to be two manifestations of the $u\bar{d}$ state.

4 The light σ resonance

A light scalar-isoscalar meson (the σ), with a mass of twice the constituent u, d quark mass coupling strongly to $\pi\pi$ is of importance in most models for

Table 2: The pole positions of the same resonances as in Table 1. The last entry is an image pole of the $a_0(980)$, which in an improved fit could represent the $a_0(1450)$. The $f_0(1300)$ and $K_0^*(1430)$ poles appear simultaneously on two sheets since the $\eta\eta$ and the $K\eta$ couplings, respectively, nearly vanish. The mixing angle δ_S for the $f_0(\approx 500)$ or σ is with respect to $u\bar{u} + d\bar{d}$, while for the two heavier f_0 's it is with respect to $s\bar{s}$.

resonance	$s_{pole}^{1/2}$	$[Res_{pole}]^{1/2}$	$\frac{-\text{Im } s_{pole}}{m_{pole}}$	$\delta_{S,pole}$	Sheet
$f_0(\approx 500)$	$470 - i250$	397	590	$(-3.4 + i1.5)^\circ$	II
$f_0(980)$	$1006 - i17$	1006	34	$(0.4 + i39)^\circ$	II
$f_0(1300)$	$1214 - i168$	1202	338	$(-36 + i2)^\circ$	III,V
$K_0^*(1430)$	$1450 - i160$	1441	320	-	II,III
$a_0(980)$	$1094 - i145$	1084	270	-	II
$a_0(1450)?$	$1592 - i284$	1566	578	-	III

spontaneous breaking of chiral symmetry, and for our understanding of all hadron masses. Thus most of the nucleon mass is believed to be generated by its coupling to the σ , which acts like an effective Higgs-like boson for the hadron spectrum. However, the lightest well established mesons in the 1994 Review of Particle Properties with the quantum numbers of the σ , the $f_0(980)$ and $f_0(1300)$ did not have the right properties. They are both too narrow, $f_0(980)$ couples mainly to $K\bar{K}$, and $f_0(1300)$ is too heavy.

The important pole in $u\bar{u} + d\bar{d}$ turns out to be the first pole in Table 2, which is the long sought for $\sigma = f_0(\approx 500)$. It gives rise to a very broad Breit-Wigner-like background, dominating $\pi\pi$ amplitudes below 900 MeV. It has the right mass and width and large $\pi\pi$ coupling as predicted by the σ model.

The existence of this meson becomes evident if one studies the $u\bar{u} + d\bar{d}$ channel separately. This can be done within the model, preserving unitarity and analyticity, by sending the s quark (and K , η etc.) mass to infinity. Thereby one eliminates the influence from $s\bar{s}$ and $K\bar{K}$ channels, which perturb $\pi\pi$ scattering very little through mixing below 900 MeV. The $u\bar{u} + d\bar{d}$ channel is then seen to be dominated by the sigma below 900 MeV.

Isgur and Speth⁴ have criticised this result claiming that crossed channel exchanges, in particular ρ exchange, is important. Our reply⁴ to this is that because of the well known result from dual models, that a sum of s -channel resonances also describe t -channel phenomena, there is no inconsistency. A resonance amplitude is always a *product* of a resonance pole and crossed channel

singularities. In the model of Ref. ^{1,2} the crossed channel singularities were, in principle, included through the form factor $F(s)$. Improvements to the model can be done by allowing for a more complicated analytic form for $F(s)$, which then furthermore can be constrained by data on exotic channels like $\pi^+\pi^+$ and $K^+\pi^+$ scattering.

Recently we became aware of three references⁵, which in addition to those given in Ref. ² also support the existence of a light σ resonance, although within more limited models. The evidence for the σ is thus mounting, and the PDG tables of 1996 have now included it. Clearly this resonance is very important for the understanding of the hadron spectrum as a whole.

5 Concluding remarks

One could argue that the two states $f_0(980)$ and $a_0(980)$ are a kind of $K\bar{K}$ bound states (c.f. Ref.⁶), since these have a large component of $K\bar{K}$ in their wave functions. However, the dynamics of these states is quite different from that of normal two-hadron bound states. If one wants to consider them as $K\bar{K}$ bound states, it is the $K\bar{K} \rightarrow s\bar{s} \rightarrow K\bar{K}$ interaction which creates their binding energy. Thus, although they may spend most of their time as $K\bar{K}$ they owe their existence to the $s\bar{s}$ state. Therefore it is more natural to consider the $f_0(980)$ and $f_0(1300)$ as two manifestations of the same $s\bar{s}$ state.

References

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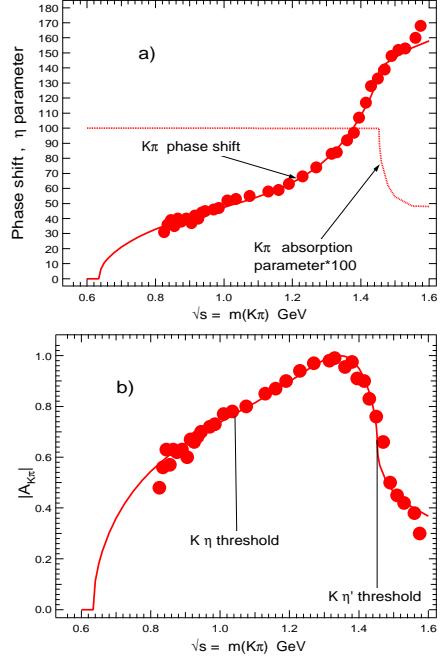


Figure 1: (a) The $K\pi$ S-wave phase shift and (b) the magnitude of the $K\pi$ partial wave amplitude compared with the model predictions, which fix 4 (γ , $m_0 + m_s$, k_0 and $s_{A,K\pi}$) of the 6 parameters.

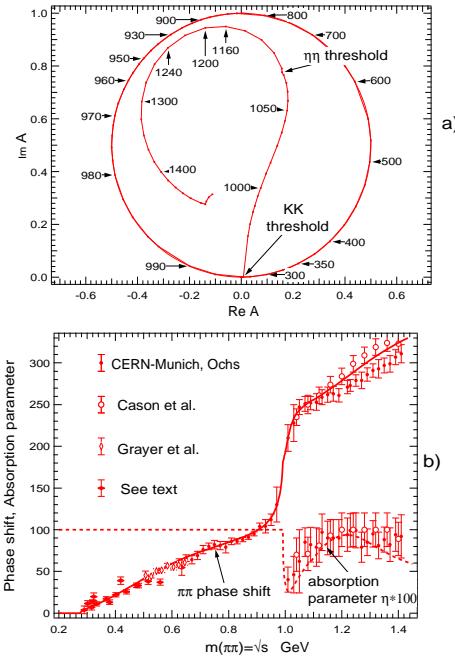


Figure 2: (a) The $\pi\pi$ Argand diagram and (b) phase shift predictions are compared with data. Note that most of the parameters were fixed by the data in Fig.1. For more details see Ref. ^{1,2}.

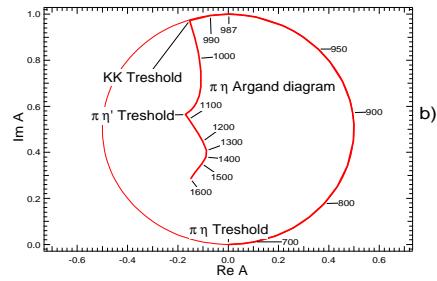
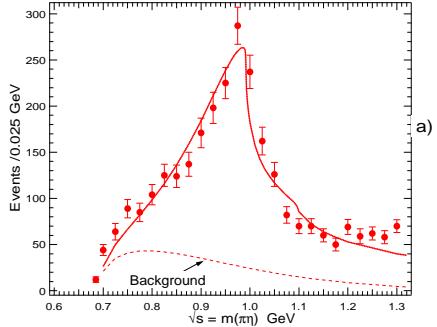


Figure 3: (a) The $a_0(980)$ peak compared with model prediction and (b) the predicted $\pi\eta$ Argand diagram

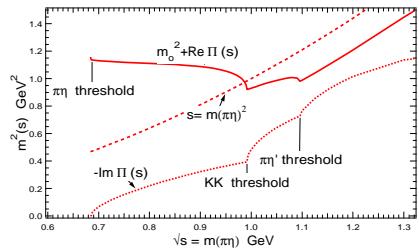


Figure 4: The running mass $m_0 + \text{Re}\Pi(s)$ and $\text{Im}\Pi(s)$ of the $a_0(980)$. The strongly dropping running mass at the $a_0(980)$ position, below the $K\bar{K}$ threshold contributes to the narrow shape of the peak in Fig. 3a.

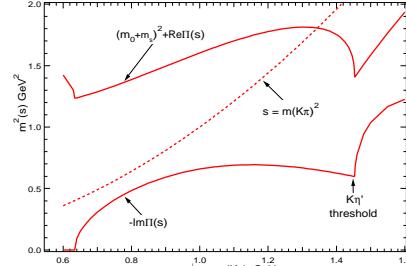


Figure 5: The running mass and width-like function $\text{Im}\Pi(s)$ for the $K_0^*(1430)$. The crossing of s with the running mass gives the 90° phase shift mass, which roughly corresponds to a naive Breit-Wigner mass, where the running mass is put constant.

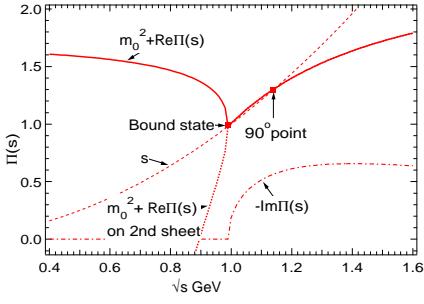


Figure 6: (a) Although the model has only one bare $s\bar{s}$ resonance, when unitarized it can give rise to two crossings with the running mass in the $s\bar{s} - K\bar{K}$ channels. This means the $s\bar{s}$ state can manifest itself in two physical resonances, one at threshold and one near 1200 MeV (See Ref. 2 for details) as in this figure.